TORSIONAL-FLEXURAL BUCKLING OF BEAMS AND COLUMNS BY APPROXIMATE METHODS OF ANALYSIS

BY

MOUNIR BADIR

B.S., Fouad I University, 1943
M.S., University of Illinois, 1947

AN ABSTRACT OF A THESIS

SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
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Introduction

Beams and columns of open section are commonly used in engineering and aeroplane structures. Their use results in an appreciable saving in weight, and renders erection, inspection and maintenance more conveniently accomplished. For such sections, it is necessary to consider the possibility of failure by torsional-flexural buckling as well as by flexural buckling, local failure or by plastic deformation when the yield point is reached.

In the theory of torsional-flexural buckling of bars of open section it has been assumed that cross-sections preserve their original shape in the buckled configuration. This assumption appears reasonable when the cross-section walls are thick enough to resist any tendencies to deform. As the walls become thinner it is expected that the cross-sections will no longer preserve their original shape, but instead, they will deform as the buckling configuration is reached. Also, as the cross-section walls become extremely thin, local failure of the bar may be anticipated at a critical stress smaller than that necessary to produce torsional-flexural buckling.

Part of the object of this thesis is to investigate the effect of the deformation of the web on the torsional buckling strength of centrally loaded I-section columns with thin webs, and to evaluate the critical stress necessary to produce local failure.

Another object of this thesis is to develop an approximate numerical procedure for evaluating the torsional-flexural buckling load for beams and columns of the open type of any shape, with any combination of end conditions and loads.

The problem of torsional-flexural buckling of beams and columns has been solved in some special cases, in general, by analytical methods. The available solutions are therefore restricted to the cases in which the analytical solutions of the differential equations of buckling are obtainable. In the general problem of torsional-flexural buckling the differential equations are so complex that an attempt to determine their analytical solution is rather impractical. The solution of these differential equations is best obtained by a numerical procedure. In contrast with the analytical methods a numerical procedure not only has the advantage of dealing with a great variety of problems under general conditions, but also renders the action of the member during buckling more clearly visualized.

Effect Of Web Deformation On Buckling Of I-Section Columns

The effect of web deformation on the torsional buckling strength of centrally loaded I-section columns with thin webs is studied by energy methods and the calculus of variations. Two apporximate forms of deformation of the web of the twisted column are assumed. They satisfy the differential equations of equilibrium of the web when the effect of Poisson's ratio is neglected. However, they fail to satisfy the boundary conditions along both edges of the buckled web. The correct form of the deformation of the web is not used because it yields results which are so complex as to be entirely impractical. The end conditions are such that both ends of the column are freely supported but the end cross-sections are not permitted to rotate about the axis of the column.

The work done by the external forces, and the strain energy of non-uniform torsion of the bar and of bending of the web, as the column deforms from its original straight pattern to the assumed buckled configuration, are determined. The condition that the above mentioned work and strain energy are equal gives the smallest load required to produce

instability of the column.

1. Buckling by Flange Twist

In this case, after the cross-sections twist about the axis of the column, the flanges undergo additional twist about their own centroids. Each flange twists through the same angle, but in opposite sense of rotation. The web accordingly deforms into a half-wave which is assumed a parabola.

The investigation leads to the following possibilities: (1) buckling by twist of the two flanges about their own centroids accompanied by deformation of the web into a half-wave, without rotation of the cross-section as a whole. The column assumes m sinusoidal half-waves along its length. This type of instability is called "buckling by flange twist" for distinction. For sections with very thin webs, it is found to occur at a load which may be smaller than Wagner's load especially in the case of short columns; and (2) buckling by twist of the cross-section as a whole not accompanied by web deformation, in other words, a deformation identical with that of Wagner's theory of torsional buckling.

2. Buckling by Torsion and Web Deformation

The form of web deformation assumed here differs from the one described above in that the twist of both flanges about their centroids occurs in the same sense of rotation and the flanges remain parallel. The web accordingly deforms into a complete wave which is assumed a cubic curve.

The investigation leads to the possibility of buckling by torsion combined with the above described form of web deformation. The buckling load is smaller than Wagner's load but the difference is inappreciable for columns of cross-sections with proportions which are commonly used in engineering practice.

The General Differential Equations For Torsional-Flexural Buckling

The general problem of torsional-flexural buckling of

beams and columns of open section is one of three degrees of freedom. During buckling, the position of any cross-section can be completely defined by three quantities, namely: translations in two directions and a rotation. With the member in a displaced and twisted position the moments of the external forces at any cross-section are determined and the three differential equations of equilibrium are written. The hypothesis made by Wagner that the shear stresses in non-uniform torsion are effectively those of uniform torsion is adopted in the derivation of these equations. The theorem of stationary potential energy is applied to obtain the general differential equations for torsional-flexural buckling from the differential equations of equilibrium. The differential equations for buckling in any special case can easily be obtained from the general differential equations for torsional-flexural buckling.

Investigation of the Stability of Centrally Loaded Columns

Instability of centrally loaded I-section columns by torsional buckling is found impossible to occur if the width of the flange is equal to or smaller than the depth of the section, except when flexural buckling is prevented in the weak direction. Curves which may be conveniently used to detect the possibility of failure of steel I-section columns by torsional buckling are included in the thesis. Other curves are constructed to demonstrate the significance of torsional-flexural buckling of channel and angle section columns.

Numerical Procedure for Torsional-Flexural Buckling of Beams and Columns of Open Section

A general approximate numerical procedure for evaluating the torsional-flexural buckling load for beams and columns with constant or variable cross-section of open type of any shape, with continuous or discontinuous axes, with various end conditions and subjected to a general type of loading, is

described and illustrated by several examples. The procedure is substantially of the same nature as one described by N. M. Newmark in the Transactions of the American Society of Civil Engineers, 1943, for computing deflections, moments and buckling loads. It is a combination of successive approximations and step-by-step procedures.

In general, the solution is started by guessing at the translation and the rotation at a number of division points on the member. With the member in its deflected configuration the moments produced by the external forces, including the assumed value of the critical buckling load, are computed. These moments, in turn, produce translations and rotations of the member, which can be obtained by using the differential equations for torsional-flexural buckling and satisfying the end conditions. In this calculation, the translations are determined by use of successive approximations procedure, consistent with the last derived rotations. Then the rotations are determined by a step-by-step process, statring at one end of the bar, consistent with the last derived translations. The assumed and the resulting quantities are compared at every division point. When they agree the computation is discontinued and the assumed load is the true critical buckling load. On the other hand, if no agreement is obtained between the assumed and the resulting quantities the sequence of the computations is further repeated after making new assumptions until the desired agreement between the assumed and the resulting quantities is obtained. The results of each cycle of the computations are used to arrive at a better assumption of the translations, rotations and the critical buckling load for the consequent cycle.

The convergence of the procedure is rapid. In most cases two or three cycles of iteration are necessary to attain convergence to three significant figures. Only 8 to 12 divisions are required to obtain an answer with high degree of accuracy. In problems of bars with constant cross-section, an accuracy can easily be obtained within 1 percent. In problems of bars with variable cross-section the accuracy of the solution is esti-

mated as within 2 percent of the so called "exact" answer in the problems solved.

Conclusions

The results of the investigation made in this thesis justify conclusions which can be briefly summarized as follows:

1. The effect of web deformation on the torsional buckling strength of centrally loaded I-section columns with thin webs is negligible for cross-sections with proportions which are commonly used in engineering practice.

2. Centrally loaded I-section columns with thin webs are more likely to buckle by flange twist, where the flanges twist about their own centroids and the web accordingly deforms into a half-wave, without rotation of the cross-section as a whole. The column assumes m sinusodial half-waves along its length.

3. Instability of centrally loaded I-section columns by torsional buckling is impossible if the width of the flange is equal to or smaller than the depth of the section, except when flexural buckling is prevented in the weak direction.

4. Th described general numerical procedure for evaluating the torsional-flexural buckling load for beams and columns is a successful one. It deals with a great variety of problems of beams and columns of constant or variable cross-section of open type of any shape, with continuous or discontinuous axes, with general end conditions and any combination of loads. The convergence of the procedure is rapid and the results obtained have a high degree of accuracy.

VITA

Mounir Badir who was born on December 10, 1922 is a citizen of Egypt. In June 1943 he obtained the degree of Bachelor of Science with honors in Civil Engineering from Fouad I University, Cairo, Egypt. He then worked for a period of three months as assistant engineer, Engineering Department, Egyptian State Railways, Ministry of Communications. In October 1943 he was transferred as demonstrator on the staff of the College of Engineering, Farouk I University, Cairo, Egypt, where he remained engaged until September 1945. At this time he was selected by the Ministry of Education and sent to the United States of America to study for the degree of Doctor of Philosophy and acquire practical experience in the field of civil engineering. He is on leave of absence from the staff of the College of Engineering, Farouk I University.

He began his graduate study in February 1946. During 1946 he wrote a master's thesis on "The Effect Of The Rivet Pattern On The Strength Of Riveted Joints." In February 1947 he obtained the degree of Master of Science in Civil Engineering. He has continued his graduate studies since that time.

In April 1947 he was elected an associate member of the Illinois Chapter of the Society of the Sigma Xi. In March 1948 he was elected a member of the Pi Mu Epsilon Fraternity, Illinois Alpha chapter, University of Illinois.